Key

Chapter 2.4 Complex numbers

Objectives: Define complex numbers and their properties

Imaginary Numbers

The imaginary unit is defined as $i = \sqrt{-1}$

If r > 0, then the imaginary number $\sqrt{-r}$ is defined as follows:

$\sqrt{r} \cdot \sqrt{-1} = i\sqrt{r}$

ex. $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = 2i$ $\sqrt{-10} = \sqrt{10} \cdot \sqrt{-1} = \sqrt{10}i$

Summary of Numbers

Complex Numbers

Real Numbers: $-5,\ -\sqrt{3},\ 0,\ \sqrt{5},\ \frac{8}{3},\ 9$ **Rational Numbers:** $-5,\ 0,\ \frac{8}{3},\ 9$ **Integers:** $-5,\ 0,\ 9$ **Whole Numbers:** $0,\ 9$ **Natural Numbers:** 9 **Irrational Numbers:** $-\sqrt{3},\ \sqrt{5}$	**Imaginary Numbers:** $-4i,\ 3+2i,\ 2i\sqrt{2}$

Example 1 isolate x^2 & $\sqrt{}$ both sides

Solve $2x^2 + 11 = -37$.

$2x^2 = -48$

$x^2 = -24$

$x = \pm\sqrt{-24}$

$x = \pm\sqrt{6}\sqrt{4}\sqrt{-1}$

$= \pm 2\sqrt{6}\,i$

Do now

$x^2 = -16$.

$\sqrt{x^2} = \sqrt{-16}$

$\sqrt{16} \cdot \sqrt{-1}$

$x = \pm 4i$

$x^2 - 8 = -36$.

$x^2 = -28$

$x = \pm\sqrt{-28}$

$x = \pm\sqrt{4} \cdot \sqrt{7} \cdot \sqrt{-1}$

$x = \pm 2\sqrt{7}\,i$

A complex number is any number that can be written as a + bi, where a and b are real numbers and i is the imaginary part

a is called the real part and **b** is called the imaginary part

example: $4+2i$ $\quad -3i$ $\quad 4 \rightarrow 4+0i$

R I $\quad 0+3i$

Example 2 Find each sum or difference. combine like terms

a) $(4-7i)+(-11+9i)$

$-7+2i$

b) $(6+8i)-(2-5i)$ Dist. -

$6+8i-2+5i$

$4+13i$

Powers of i

$i = i$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = 1$

Example 3 Multiply

a. $-2.5(8-9i)$

$-20+22.5i$

b. $(3-2i)(3+2i)$ FOIL change $i^2=-1$

$9-6i+6i-4i^2$

$9-4(-1)$

$9+4$

13

c. $(2-i)(-3-4i)$ FOIL

$6-8i+3i+4i^2$

$6-5i+4(-1)$

$6-4-5i$

$2-5i$

DO NOW Practice add

1) $(-4+2i)+(6-3i)$

$2-i$

FOIL Dist.

2) $(3-2i)^2-6(3-2i)+13$

$(3-2i)(3-2i)$

$9-6i-6i+4i^2-18+12i+13$

$0i$

$9+4(-1)-18+13=0$

☺

Key

Conjugate of a Complex Number

The conjugate of a complex number a + bi is $a-bi$

change signs of imaginary term

Find the conjugates.

a. 7-8i $7+8i$

b. -2+ i $-2-i$

c. 4-5i $4+5i$

Example 4 Division with complex numbers

Simplify $\frac{3-2i}{-4+i}$. Write your answer in standard form.

multiply by the conjugate of the denominator

$$\frac{3-2i}{-4+i}\cdot\frac{-4-i}{-4-i}=\frac{-12-3i+8i+2i^2}{16-i^2}$$

$$=\frac{-12+5i+2(-1)}{16-(-1)}=\frac{-14+5i}{17}=\boxed{\frac{-14}{17}+\frac{5i}{17}}$$

Do Now Practice

Simplify $\frac{4-3i}{-1+2i}$. Write your answer in standard form.

$$\frac{4-3i}{-1+2i}\cdot\frac{-1-2i}{-1-2i}=\frac{-4-8i+3i+6i^2}{1-4i^2}=\frac{-4-5i+6(-1)}{1-4(-1)}=\frac{-10-5i}{5}=\boxed{-2-i}$$

Example 5 Factor the sum of squares

x^2+y^2

$\sqrt{-y^2}=\pm yi$

$x^2-(-y^2)$

$\boxed{(x+yi)(x-yi)}$

$(-y^2)\rightarrow$

$12x^2+3$

$3(4x^2+1)$

$\sqrt{-1}=\pm i$

$3(4x^2-(-1))$

$\boxed{3(2x+i)(2x-i)}$